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Monitoring of Geostationary Earth Orbit Satellites in Russian Space Surveillance Center

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Cataloging of geostationary-Earth-orbit (GEO) satellites commenced in the Russian Space Surveillance Center (SSC) in the 1980s. Enhancement of the software tools for catalog maintenance has continued since that time. The current status of this software complex is described. The characteristics of the sensors and the observed satellites that determine the structure of the catalog maintenance algorithms are discussed. There are certain essential limitations of the range of the observed longitudes of GEO satellites, the frequency of the observations, and their accuracy. Under these conditions, the loss of measurement data is unacceptable; the algorithms must use any observational data for catalog maintenance. The correlation of measurements with the cataloged orbits and subsequent orbit updating must be possible for the very wide range of the accuracy of the measurements. A fast and accurate prediction algorithm was developed to solve this task successfully. Thus, it became possible to develop and implement in the SSC an efficient complex of software tools for catalog maintenance. The general scheme is described of this complex and of its most significant components: primary determination of orbits, correlation of the observations with the cataloged satellites, updating of orbits using the measurements, planning of the observations and targeting of the sensors, and preliminary tracking of new satellites. Some results of the testing, based on real data, are presented.

Introduction

REFERENCES 1 and 2 describe the general structure, characteristics, and the algorithms for maintenance of the catalog of low-Earth-orbit (LEO) (perigee altitude less than 300 km) satellites in the Russian Space Surveillance Center (SSC). This paper treats the same issues for satellites in geostationary Earth orbit (GEO). The domain of the phase space will be considered geostationary when the orbital parameters satisfy the conditions

$$1 - \delta T < T < 1 + \delta T, \qquad 0 \le e < \delta e, \qquad 0 \le i < \delta i \quad (1)$$

where T, i, and e are the orbital period, inclination, and eccentricity, respectively, and δT, δi, and δe are constants. We choose $\delta T = 0.125$ days, $\delta i = 30$ deg, and $\delta e = 0.2$.

First we analyze the initial background data, describing the conditions for catalog maintenance. These data include the characteristics of the sensors and the characteristics of satellite motion. Then the general scheme and its components are described, in particular, the algorithm for primary determination of orbits (i.e., estimation of the satellite parameters using one measurement and a priori data), the algorithm for assigning the measurements to cataloged satellites (i.e., the procedure, making the decision of assigning certain measurements to cataloged satellites), the algorithm for updating the orbits using the data of the measurements, the procedure for planning the observations and calculating the target indications for the sensors, and also the process of preliminary tracking of new satellites that arrived in the catalog (i.e., the tracking of the satellites that are not correlated satellites cataloged by the SSC immediately after their arrival).

In the course of the study, we compare the resulting procedures with the process of maintenance of LEO satellite catalog and analyze the observed difference.

Sensors and Measurements

The basic source of data for cataloging LEO satellites are the detection radars.[3] These are the designated sensors of the ballistic missile warning system and BMD system, used for space surveillance purposes.

Currently, Russia does not have designated sensors capable of monitoring the geostationary belt. To solve this task, the SSC uses optical sensors. Their location, type, and affiliation are given in Table 1.

Let us consider the major features of this network.

1) The sensors of the network do not provide complete coverage of the GEO ring. The monitored longitudes of satellite projections (the point on the surface of the Earth where the straight line connecting the position of the satellite in space and the center of the Earth crosses it) range from ≈30° W to ≈160° E, that is, about one-half of the ring.

2) The detection radars continuously monitor the space within their fields of view. The optical stations cannot operate like this. They are subjected to certain limitations related to meteorological conditions and positions of the sun and the moon and also to the

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Table 1 Optical sensors

Location	Type	Affiliation
Uzhgorod (Ukraine); ≈49° N, ≈22° E	Photography	Uzhgorod State University
Simeiz (Ukraine); ≈44° N, ≈34° E	Photography, optical–electronic	Institute of Astronomy, Russian Academy of Sciences
Zvenigorod (Russia); ≈56° N, ≈37° E	Photography	Institute of Astronomy, Russian Academy of Sciences
Zelenchuk (Russia); ≈41° N, ≈44° E	Optical–electronic	Scientific Center "Cosmoten"
Ashkhabad (Turkmenistan); ≈38° N, ≈58° E	Optical–electronic	Astronomical Center "Asman"
Kourovka (Russia); ≈57° N, ≈60° E	Photography, optical–electronic	Urals State University
Dushanbe (Tadzhikistan); ≈38° N, ≈69° E	Photography, optical–electronic	Institute of Astrophysics, Tadzhikistan Academy of Sciences
Alma-Ata (Kazakhstan); ≈43° N, ≈77° E	Optical–electronic	Astrophysics Institute of Kazakhstan Academy of Sciences
Mondy (Russia); ≈52° N, ≈101° E	Optical–electronic	Institute of Sun and Earth Physics Siberian Branch, Russian Academy of Sciences

limitations related to the specific planning of the observations at the sites. For optical stations, the work for the SSC is by no means their only job and very seldom has the first priority. Thus, all of the stations operate with significant gaps that may last as long as several months.

3) The optical sensors measure the angular coordinates of a satellite: the right ascension α and declination δ in the local equatorial coordinate frame. The errors of single measurements normally are within the range 1–10″. In distance values, this corresponds to the order of the errors of the range measurements by the detection radars. Thus, the accuracy of the measurements is rather high [measurment is defined as the set of single measurements acquired for one satellite at different times within one observation session of a specific sensor (the session lasts for one or several nights)]. However, in distinction from the radars, the optical sensors do not measure the range. Thus, the acquisition of the complete six-dimensional vector of the same order of accuracy, as for the radar measurements of LEO satellites, requires several single measurements (marks) over a rather long time interval (a mark is the measurement of the right ascension α and the declination δ in the topocentric equatorial coordinate frame of the sensor for a certain time t). The analysis revealed that the orbit can be determined with accuracy sufficient for the SSC when the marks belong to at least two nights and, for one of the nights, there are at least four marks uniformly separated in time within an interval not shorter than 2–3 h. This is the SSC requirement for the optical sensors' measurements. However, for several reasons, the measurements do not always satisfy this requirement. More than one-half of the measurements are acquired during only one night and do not cover the required time arc. In this case, one or two (out of six) orbital parameters cannot be determined with sufficient accuracy. (Note that one of these parameters is always the orbital period because the errors of its determination on the basis of a group of marks can reach several tens of minutes.)

4) Several optical–electronic stations (Alma-Ata, Zelenchuk, Mondy) maintain their own catalogs of the objects within their zone. The measurements acquired by these sensors are correlated there with the on-site catalogs, and when such measurements arrive, the SSC already knows to what satellite it is correlated (according to the on-site correlation). However, the majority of the optical stations (all of the photography stations and some optical–electronic) do not correlate the measurements with the catalog, and the SSC receives the measurement without reference to a satellite. The groups of measurements from these stations sometimes (normally, less than 10% of cases) include the marks produced by several (more than one) satellites.

Satellite Motion

The observed objects (artificial space bodies, moving in orbit about the Earth) are moving in space. A model of their motion (i.e., the algorithm for calculation of the orbital parameters for the time t using their values at the time t_0) is needed to design the catalog maintenance algorithm. The orbital parameters $\boldsymbol{a}$ of any space object satisfy a system of first-order differential equations of motion, thus, $\boldsymbol{a}(t) = \boldsymbol{U}[\boldsymbol{a}(t-\tau), \tau]$. The precise functional relationship $\boldsymbol{U}(\boldsymbol{a}, \tau)$ is not known, and in practice, the approximate relationships $\boldsymbol{U}_0(\boldsymbol{a}, \tau)$ used are those for the specific algorithm for prediction of the orbital parameters.

Real prediction errors $\boldsymbol{V} = \boldsymbol{a}(t) - \boldsymbol{U}_0$ result from the inexact accounting of the perturbing factors and comprise two components: systematic error $\boldsymbol{V}_1$ and the nonremoved error $\boldsymbol{V}_2$. The systematic error is the result of the inexact accounting of the perturbations. The nonremoved error originates from not accounting for certain real perturbing factors.

The three basic requirements for the prediction algorithm are as follows:

1) Real prediction errors should not make the correct correlation of measurements to cataloged satellites impossible.

2) Real prediction errors should not exceed the values of the order of 3–5′, defined by the maximum permissible errors of target indication for an optical–electronic sensor.

3) The CPU time of the computer code must be acceptable.

Because the basic operations of the algorithm (correlation of measurements with cataloged satellites and orbit updating) often must be performed involving measurements and orbits significantly distant in time (up to several years), the creation of the relevant procedure is not an easy task. However, we managed to solve it.

We will not treat this algorithm in detail here because it is described in an earlier paper[4] and Refs. 5 and 6 consider the subject as well. The basic characteristics of this algorithm are as follows:

1) Systematic errors of the prediction algorithm do not exceed 3′(≈38 km) for propagation intervals up to four years.

2) The real prediction errors for propagation intervals up to 500 days do not exceed 1′(≈12 km) for 50% of the passive satellites of the real catalog. (The "Results of Trial Operations" section treats the issue of real orbit determination and propagation errors in more detail.)

3) The computation time t_{comp} of the propagator (for Elbrus-2 computer, computation rate 3.5 million average operations per second) can be assessed using the formula

$$t_{\text{comp}}[s] = 0.003N_{st} + 0.001N_t \qquad (2)$$

where N_{st} is the number of the integration steps (the step for the integration is chosen within the range 10–20 days) and N_t is the number of the points for which the propagation is fulfilled (within the total prediction interval).

The processing of measurements often requires the propagation of all of the orbits of the catalog to a certain epoch. The following sections will illustrate that this operation simplifies the data processing. The characteristics presented allow propagation of the whole catalog for one year in 0.5–1.0 min. This is quite acceptable.

Structure of the Algorithm

The structure of the catalog maintenance algorithm in general is similar to the structure of the procedure used for LEO satellites. However, some specific features, determined by the structure of the initial data, do exist.

The measurements received by the SSC enter the procedure of primary orbit determination. For each measurement, the parameters of the orbit and the errors of their determination are estimated on the basis of the single measurements (marks) and the a priori data. Then for each measurement, we determine whether the satellite that produced it exists in the catalog, that is, the task of assigning the measurements to the cataloged satellites is solved. As a result, the measurement becomes either assigned (correlated) to a certain satellite or remains free (uncorrelated).

The correlated observations update the orbital parameters of the satellites that produced them, that is, the task of orbit updating is solved. If we manage to correlate the measurements to a certain satellite and update its orbit regularly, the satellite is monitored or tracked, that is, the process of correlating new measurements to the satellite and updating its orbit using them. When the flux of the measurements of certain object ceases or weakens significantly, the tracking of the satellite may stop. The break of tracking occurs.

The uncorrelated measurements are included into the catalog as new satellites and participate in the tracking process together with the other cataloged objects.

At the initial stage of this process, the preliminary tracking, the new satellite is identified with the objects already present in the catalog and with the lost ones. In case the satellite is identified with the previously cataloged object, the data on this satellite are renewed and its tracking is thus recovered. When the decision is made that the orbit belongs to a new, not previously, cataloged satellite, its origin (international designator) must be determined, that is, the satellite identification task must be solved.

In contrast to the catalog maintenance process for LEO satellites, the interaction with the network of optical sensors includes the feedback, that is, the sensors work under the control of the SSC. The feedback means planning of observations and calculation of target indications for the sensors. The objects for the assumed future work of any optical station are selected, and the respective target indications are calculated.

The following sections describe the basic algorithms used for maintenance of the catalog of GEO satellites in more detail.

Primary Determination of Orbits

The detailed consideration and analysis of the algorithm for primary determination of orbits is given in Ref. 4. Because many upgrades have been introduced into this procedure in the course of the past 10 years, we will give a new description here.

The estimation of the orbital parameters is the point of the minimum of the functional

$$\varphi(\boldsymbol{a}) = \sum_{p=1}^{r} \left\{\omega_{\alpha p}[\alpha_p - \alpha_p(\boldsymbol{a})]^2 + \omega_{\delta p}[\delta_p - \delta_p(\boldsymbol{a})]^2\right\} + \omega_{a1}(a_1 - a_{1a})^2 + \omega_{a2}(a_2 - a_{2a})^2 + \omega_{a3}(a_3 - a_{3a})^2 + \omega_{a4}(a_4 - a_{4a})^2 + \omega_{a5}(a_5 - a_{5a})^2 + \omega_{a6}(a_6 - a_{6a})^2 \tag{3}$$

where (α_1, δ_1), $(\alpha_2, \delta_2), \ldots, (\alpha_r, \delta_r)$ are the marks of the measurement with time references $t_1 \le t_2 \le \cdots \le t_r$; $\alpha_p(\boldsymbol{a})$ and $\delta_p(\boldsymbol{a})$ are the functional relationships between parameters α_p and δ_p of pth mark and parameters $\boldsymbol{a} = (a_1, a_2, \ldots, a_6)$ of the satellite; $\omega_{\alpha p}$ and $\omega_{\delta p}$ are the weights of the components of pth mark (the values inversely proportional to the variances of the components' errors); $\boldsymbol{a}_a = (a_{1a}, a_{2a}, \ldots, a_{6a})$ is the vector of a priori values of orbital parameters; and $\omega_{a1}, \omega_{a2}, \ldots, \omega_{a6}$ are the weights of the components of $\boldsymbol{a}_a$ (the values are inversely proportional to the variances of the errors of these components).

For the vector $\boldsymbol{a}$ of orbital parameters, we take the six-dimensional vector of the elements $\boldsymbol{a} = (\tilde{\lambda}, L, p, q, \tilde{h}, \tilde{k})$, derived from the known Keplerian elements M, a, i, e, Ω, and ω using the formulas

$$\tilde{\lambda} = M + \omega + \Omega, \qquad L = \sqrt{\mu a}, \qquad p = \sin(i/2)\cos\Omega$$

$$q = \sin(i/2)\sin\Omega, \qquad \tilde{h} = e\sin(\omega + \Omega), \qquad \tilde{k} = e\cos(\omega + \Omega)$$

where μ is the gravitational constant. Vector $\boldsymbol{a}$ is referred to the certain time t_m $(t_1 \le t_m \le t_r)$.

The components of the a priori vector $\boldsymbol{a}_a$ are the average expected values for GEO satellites.

Next we describe the organization of the computations, the minimization of the functional, and the construction of the initial approximation.

Organization of the Computations

The marks of the measurement are tested for reliability using the a priori range of the possible variation of the parameters and the smoothness. The marks considered unreliable are sorted out. When the primary orbit is determined, these marks are again analyzed. In case the rough errors can somehow (either obviously or using the repeated request for the sensor) be corrected, the corrections are made, and the primary orbit is recalculated.

In case the epoch to which the coordinates of the measurements are referred to does not coincide with the epoch of the measurement, the construction of the primary orbit includes the transformation of the angular coordinates to the epoch of the measurement.

The initial approximation used in the minimization of the functional $\varphi(\boldsymbol{a})$ is then constructed and this algorithm is described in the "Determination of Initial Approximation" section. Then the minimum of $\varphi(\boldsymbol{a})$ is sought. The minimization uses the multipass scheme, selecting during each pass the abnormal components of all of the marks using the normalized residuals between the measured and assessed values. For the first pass, all of the marks are taken with the weights calculated according to the variances of the errors. Before each next pass, the components of the marks are tested for abnormality by comparing to the threshold c_a, the squared normalized residual [the respective component in $\varphi(\boldsymbol{a})$] for the point of the minimum, obtained for the previous pass. The weights of the discovered abnormal components are set to zero, and the normal components of the marks, mistakenly excluded from the previous processing, are reintroduced into the process by the recovery of their weights. When the minimization shows that the weights are chosen correctly, the pass is completed. The algorithm for the minimization of $\varphi(\boldsymbol{a})$ used at each pass is described in the "Minimization of the Functional" section.

If the completed cycle of minimizations yields the decision (see the "Minimization of the Functional" section), its reliability is tested. The decision is considered reliable if both components of at least three marks have completely inscribed into the determined orbit (in case the measurement comprises only two marks, both of them must inscribe into the orbit) and the share of the marks acquired during each night, for which both components have inscribed into the orbit, exceeds the specified threshold, depending on the number of the marks for this night.

This scheme of the computation process is close to the scheme used for the tracking of LEO satellites.[2] However, the specific features of the task require a somewhat more sophisticated scheme. The number of the marks is often small, and the presence of even one abnormal essentially results in its selection. Therefore, the following additional operations are performed.

First, in the beginning of the minimization cycle, the high threshold $c_a = 10{,}000$ is used for selection of abnormal components. If this threshold results in the successful selection of abnormal components and a reliable solution is obtained, the threshold is decreased 10 times and the new minimization cycle is performed, etc. The last minimization cycle uses $c_a = 10$.

Second, when the described scheme does not lead to a reliable solution, we try to obtain the solution using all of the marks but one. The marks of the measurement are sequentially excluded from the processing. The procedure continues until a reliable solution is obtained.

If we still do not manage to obtain a reliable orbit, the analysis of the initial data and the process of computation are completed. Sometimes the changes of the parameters of the procedure or the splitting of the set of the marks into specific parts, used separately for orbit determination, yields the reliable solution.

The most frequent situations are the following two:

1) An orbital correction occurred within the interval of observations. The analysis of the residuals between the determined orbit and the marks, parameters of the orbit, and the parameters of the marks reveal such a situation. In this case, we try to determine the orbit using the marks of each night separately.

2) The observed object has an orbit with significant eccentricity (normally greater than 0.1), and the failure to obtain the reliable solution is caused by the incorrect choice of the weights ω_{a5} and ω_{a6} in Eq. (3). The analysis of the components $\varphi(\boldsymbol{a})$ and the parameters of the determined orbit can reveal this situation. In this case, the weights ω_{a5} and ω_{a6} are changed (decreased), and the primary orbit is calculated once more.

Minimization of the Functional

The algorithm was developed by Boikov. This algorithm is also used for the maintenance of the LEO satellite catalog.[2] A detailed analysis of the performance of this algorithm is given in Ref. 4. Here we present only a brief description.

A combination of two classical minimization techniques, the Newton–Gauss technique and the fastest descent technique, designed specifically for the determination of orbits on the basis of measurements (of rather general structure) is used in the algorithm.

The algorithm has three levels (0, 1 and 2). Each of the levels uses its own technique for determination of the step. The transitions between the levels are performed following the scheme $0 \to 1 \to 2 \to 0 \to \cdots$. Transition to the higher level is performed when no significant (more than 10%) decrease of the functional was achieved at the current step.

After one step of the second level, the transition to the zero level always takes place (disregarding the relationship between the initial and the final values of the functional).

Iteration always starts from the zero level. Here either the Gauss–Newton or the fastest descent (according to the step of Gauss–Newton) technique is chosen. The descent is chosen when the normalized step exceeds the threshold value. (Note that each of the parameters is normalized by the maximal expected value of the error of determination of this parameter in initial approximation.) Otherwise, the Gauss–Newton step is performed. In case the descent is chosen, three steps are always performed.

At the first level, the Gauss–Newton step is always used.

At the second level, a Gauss–Newton step is performed followed by three descents (corrective descents).

The investigations revealed that for the task under consideration, correction of the step of Gauss–Newton by several sequential descents often assists the convergence when Gauss–Newton and the fastest descent techniques fail to be efficient. In this situation, the corrective descents play the following role. The level curves of the functional have ravine structure and the long step of Gauss–Newton leads us out of the unfortunate domain. This step switches the process to the other side of the ravine from which we can descend more efficiently.

At all of the levels, the magnitude of any step in the chosen direction is determined using the minimization along this direction. This one-dimensional minimization is performed in several iterations. The process is as follows.

The evolution of the functional in the chosen direction is approximated by a third-order polynomial. The polynomial is constructed using four points: the values of the functional and its derivative along this direction in the beginning and at the end of the calculated step (of the descent or Gauss–Newton). The point of the minimum of the polynomial is determined within the interval of the step. The algorithm is rather simple and, in fact, can be reduced to solving the quadratic equation.

The experiments performed demonstrated that rather often the abrupt change of the functional along the chosen direction cannot be approximated by the third-order polynomial. Therefore, the point of the minimum is specified iteratively using the polynomial constructed for a smaller interval. For the next iteration, the beginning of the interval remains the same, and the end is the point of the minimum of the functional for the current iteration. No more than three such iterations are performed. The iterations stop when the minimum is achieved at one of the ends of the interval.

The iterations for minimization of $\varphi(\boldsymbol{a})$ are completed in one of the following five cases:

1) The value of the normalized step of Gauss–Newton became smaller than the threshold value and still the relative variation of the functional does not exceed a specified value (we arrived at the point of the minimum).

2) A specified number of iterations is completed and the absolute value of the functional is lower than the threshold value (the orbit, inscribing into all of the marks, is obtained).

3) A specified maximum number of iterations have been performed, but conditions 1 or 2 are not satisfied (divergence).

4) For the fourth time, we enter the second level (we got stuck).

5) In the process of the iterations, the values of the parameters have significantly deviated from the a priori values, and the calculated values of some of the parameters are outside the interval of physical limits (we have lost the right way).

Only in the first two cases do we consider that the solution is obtained.

Calculation of the matrix of partial derivatives of the parameters of the observation with respect to orbital parameters needed to determine the step is fulfilled using analytical formulas, taking into account the second zonal harmonic of the geopotential and perturbations from the moon and the sun. When the point for which they must be calculated is changed, the matrices are recalculated. A special autonomous mode for numerical calculation of the partial derivative matrices also exists.

Determination of the Initial Approximation

The iterative process of the minimization of the functional $\varphi(\boldsymbol{a})$ starts with the initial approximation of the orbital parameters vector $\boldsymbol{a}_0$. The convergence of the iterative process, described in the "Minimization of the Functional" section, depends on the accuracy of $\boldsymbol{a}_0$.

In the primary determination of the orbit, the satellite corresponding to the measurement is usually not known, and thus the design of the algorithm for determination of the initial approximation of good quality is an original task.

The algorithm was developed by Boikov. This is the modification of the known Laplace technique,[7] which in its pure form is inefficient due to high sensitivity to the errors in the initial data.

The essence of the improvement is to gain the advantage from the a priori data on the orbital parameters. Our algorithm uses only the a priori data on the semimajor axis, that is, it is considered that $a = a_0$, where a_0 is constant, equal to the semimajor axis of the GEO satellite with an orbital period equal to 1 day. Thus, the developed technique can be used not only for the objects within geostationary ring, but also for other important types of objects, for example, for satellites with half-day orbital period.

The derivation of the formulas for the case $p \geq 3$, the computational aspects, and the analysis of the performance of the procedure (for real measurements on various classes of satellites) are presented in Ref. 4. Here we present only a brief description.

The vectors $\boldsymbol{r} = (X, Y, Z)$ and $\dot{\boldsymbol{r}} = (\dot{X}, \dot{Y}, \dot{Z})$ of geocentric positions and velocities, used for the determination of the orbital elements of the initial approximation, are calculated using the equations

$$\boldsymbol{r} = D\boldsymbol{D}_0 + \boldsymbol{r}_s, \qquad \dot{\boldsymbol{r}} = \dot{D}\boldsymbol{D}_0 + D\dot{\boldsymbol{D}}_0 + \dot{\boldsymbol{r}}_s \tag{4}$$

where D is the distance from the sensor to the object, $\boldsymbol{D}_0$ is the unit vector of the direction $\boldsymbol{D}$ from the location point of the sensor to the object, and $\boldsymbol{r}_s$ is the vector from the center of the Earth to the point of the location of the sensor;

The value D is the root of the equation

$$\nu^2(D) + \mu/a_0 - 2\mu/r = 0 \tag{5}$$

where r and ν are the absolute values of the vectors r and $\dot{r}$ and are calculated using Eq. (4):

$$\dot{D} = -[D(\boldsymbol{r}_s, \boldsymbol{D}_0, \ddot{\boldsymbol{D}}_0) + (\boldsymbol{r}_s, \boldsymbol{D}_0, \ddot{\boldsymbol{r}}_s)]/2(\boldsymbol{r}_s, \boldsymbol{D}_0, \dot{\boldsymbol{D}}_0) \tag{6}$$

The expression $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$ means the combined product of vectors $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$; μ is the gravitational constant; and a_0 is the a priori value of the semimajor axis. The vectors $\boldsymbol{D}_0$, $\dot{\boldsymbol{D}}_0$, and $\ddot{\boldsymbol{D}}_0$ are determined in the following way. Using the pth mark, we calculate the direction cosines vector $\boldsymbol{D}_{op}$. Then using the vectors $\boldsymbol{D}_{op}$ as the measurements, the vector coefficients of the polynomial of the second order (in time) are estimated. These are the vectors to be found.

In the case where only two marks are available ($p = 2$), parameters D_0, α_0, δ_0, $\dot{D}_0$, $\dot{\alpha}_0$, and $\dot{\delta}_0$ of the initial approximation with the reference time $t_0 = 0.5(t_1 + t_2)$ are calculated using the formulas

$$D_0 = \sqrt{D_{01}^2 + R_E^2 - 2D_{01}R_E\cos\varphi_s\cos(\lambda_s - \lambda_0)}$$

$$\alpha_0 = 0.5(\alpha_1 + \alpha_2), \qquad \dot{\alpha}_0 = (\alpha_2 - \alpha_1)/(t_2 - t_1)$$

$$\delta_0 = 0.5(\delta_1 + \delta_2), \qquad \dot{\delta}_0 = (\delta_2 - \delta_1)/(t_2 - t_1)\dot{D}_0 = 0 \tag{7}$$

where α_1, δ_1, t_1 and α_2, δ_2, t_2 are parameters of the marks and their epochs; D_{01} is the radius of circular orbit with an orbital period of one day; R_E is the average radius of the Earth; λ_s and φ_s are longitude and latitude of the location of the sensor; and λ_0 is the longitude of satellite projection with coordinates D_{00}, α_0, δ_0.

After that, they are transformed into the orbital elements $\tilde{\lambda}_0$, L_0, p_0, q_0, $\tilde{h}_0$ and $\tilde{k}_0$. In case the obtained values of L_0, $\tilde{h}_0$, and $\tilde{k}_0$ differ significantly from the a priori values, they are replaced by the latter.

When only one mark is available, the initial approximation (it plays the role of the primary determined orbit) is calculated as follows:

$$\alpha_0 = \alpha_1, \qquad \dot{\alpha}_0 = \dot{\alpha}_a, \qquad \delta_0 = \delta_1, \qquad \dot{\delta}_0 = 0, \qquad \dot{D}_0 = 0$$

$$D_0 = \sqrt{D_{01}^2 + R_E^2 - 2D_{01}R_E\cos\varphi_s\cos(\lambda_s - \lambda_0)} \tag{8}$$

where $\dot{\alpha}_a$ is the angular velocity of rotation of the Earth.

The obtained values are transformed into orbital elements $\tilde{\lambda}_0$, L_0, p_0, q_0, $\tilde{h}_0$ and $\tilde{k}_0$, and after that the values of L_0, $\tilde{h}_0$, and $\tilde{k}_0$ are replaced by the a priori values.

Measurement-to-Satellite Correlation

After determination of the primary orbits, the measurements are correlated with the cataloged satellites. All of the measurements are subjected to this procedure including those that are correlated onsite. (The on-site catalog may differ from the catalog of the SSC, and, therefore, the measurements must be correlated to the catalog once more.)

The algorithm used here differs from the algorithm used for maintenance of the catalog of LEO satellites. The basic rationale is as follows:

1) The flux of optical measurements is 3–4 orders of magnitude weaker than the flux of the radar measurements, and the number of cataloged satellites within the geostationary ring is an order of magnitude less than the number of LEO satellites.

2) The accuracy characteristics of radar and optical observations essentially differ as well. The most accurate parameter of radar observations is the range and for optical observations it is the angular coordinates.

3) The character of orbital motion of GEO and LEO satellites is also different. The most unstable atmospheric perturbations, very important for the latter, do not affect GEO satellites. On the other hand, the orbital maneuvers and corrections (which affect the tracking process) are performed by less than 0.5% of cataloged LEO objects compared to more than 30% of maneuvering GEO satellites.

The first reason allows the most accurate, but labor-consuming, techniques to be used for maintenance of the catalog of GEO satellites. The second reason leads to different decision functions for the algorithms, and the third one requires a special algorithm for GEO satellites with corrected orbits.

The accuracy characteristics of radar and optical observations vary over an extensive range, thus leading to increased sophistication of the algorithm.

Because active satellites maneuver different procedures are used to correlate the measurements to active or passive satellites. (An active satellite is one for which the evolution of orbital parameters cannot be described by the accepted model of orbital motion due to orbital corrections and maneuvers. A passive satellite is one for which the evolution of orbital parameters is satisfactorily described by the accepted model of orbital motion.) For time intervals significantly longer than the interval between the neighboring measurements, a passive cataloged satellite is characterized by one orbit, obtained using all of the observational data on the satellite acquired within this interval. This orbit contains all of the information required for making decisions regarding correlation of measurements to this satellite. For an active satellite, one orbit cannot provide sufficient information because the orbit has been changed by the maneuver. Efficient decision making for these satellites requires the knowledge of at least one orbit between the corrections and the a priori data on the character of the performed maneuver (the best is the data on the time and momentum of the maneuver).

A priori we do not know which satellite (active or passive) has produced the measurement. Thus, each measurement is subjected to correlation procedures of both types. The measurement cannot be correlated to more than one satellite of each class. When a measurement is correlated to certain active and passive satellites, the final decision on its assignment is the responsibility of the analyst.

The majority of the correlation decisions are made automatically without the interference of the analyst into the computations. However, completely automatic performance has not yet been achieved. Thus, some of the decisions are made by the analyst, working with the results of the calculations performed by the automatic program.

Hereafter the passive and active satellite cases are considered separately.

Passive Satellites

The algorithm performs the correlation of measurements to cataloged satellites that were either always passive or were active only until the time of the observation. (In case the satellite was active before the time of measurement and after that became passive, the orbit obtained using the measurements acquired during the passive interval must be in the catalog.)

Denote the parameters of the primary determined orbit, calculated on the basis of measurement $\boldsymbol{x}$, as $\tilde{\lambda}_m$, L_m, p_m, q_m, $\tilde{h}_m$, and $\tilde{k}_m$, their time t_m, and the marks and their timing α_{mj}, δ_{mj}, and t_{mj}, $j = 1, 2, \ldots, p$. The algorithm uses only one of the values characterizing the accuracy of orbital parameters, the root-mean-square deviation σ_{LM} of the error of the determination of L, which is transformed into σ_{Tm}, the root-mean-square deviation of the error of determination of orbital period T. (The correlation matrix of the errors of determination of primary orbital parameters is calculated in the process of minimization of the functional using the Gauss–Newton technique.)

When the orbital parameters are used, Kepler's elements, i_m, Ω_m, T_m, e_m, and ω_m, and the longitude of satellite projection λ_m are calculated. Parameters of all cataloged satellites, passive at the moment t_m, are propagated to the moment t_m and are transformed into the same parameters as the measurement. We will denote the result i_s, Ω_s, T_s, e_s, and ω_s. (We do not indicate the catalog number of the satellite for simplicity.) The root-mean-square deviation of the error of determination of the orbit of cataloged satellite is denoted σ_{Ts}.

For the automatic mode, the decision function is as follows:

1) Preliminary correlated satellites are sought. They satisfy the conditions

$$\Delta i = |i_m - i_s| < c_{ip}, \qquad \Delta\Omega = |\Omega_m - \Omega_s|_{\mathrm{mod}2\pi} < c_{\Omega p}/i_s$$

$$\Delta\lambda = |\lambda_m - \lambda_s|_{\mathrm{mod}2\pi} < c_{\lambda p}, \qquad \Delta T = |T_m - T_s| < c_{Tp} \tag{9}$$

where c_{ip}, $c_{\Omega p}$, $c_{\lambda p}$, and c_{Tp}, the gates, are chosen to provide a low frequency of false correlations. (The gates c_{ip}, $c_{\Omega p}$, and $c_{\lambda p}$ are constants; the gate for the period c_{Tp} depends on σ_{Tm} and σ_{Ts}.)

2) For each of the preliminary correlated satellites, we calculate the functional

$$F = \Delta i/c_{i1} + \Delta\Omega/c_{\Omega 1} + \Delta\lambda/c_{\lambda 1} \tag{10}$$

where c_{i1}, $c_{\Omega 1}$, and $c_{\lambda 1}$ are constants. The measurement is finally assigned to the preliminary correlated satellite corresponding to minimal value of F.

Active Satellites

The algorithm correlates the measurements to the satellites that were active before the time of observation. The characteristic feature of the algorithm is taking into account the data on the corrections for each satellite individually. Almost all of the active satellites perform orbital corrections to keep the satellite longitude position within certain limits (different for different objects). For some objects, the direction to the satellite from a specific point on the surface of the Earth is also kept within certain limits during required time intervals.

For the preliminary correlation of the measurement $\boldsymbol{x}$ performed at t_m, for each cataloged satellite two orbits of orbital data from the archive are used [for each satellite, the archive of orbital data includes the orbits determined using all of the measurements, correlated to this satellite and also the orbital data, acquired from other sources (data from other sensors or space monitoring systems, etc.)] for which timing t_{s1} and t_{s2} satisfy the condition timing $t_{s1} < t_m < t_{s2}$.

(In case there are no orbits satisfying $t_{sj} > t_m$ in the archive, two identical orbits are taken.)

The elements i_m, Ω_m, and T_m and the longitude of satellite projection λ_m are calculated using the measurement, and on the basis of both archived orbits, we determine parameters i_{s1}, i_{s2}, T_{s1}, T_{s2}, λ_{s1}, and λ_{s2} without propagation and parameters $\tilde{i}_{s1}$, $\tilde{i}_{s2}$, $\tilde{\Omega}_{s1}$, and $\tilde{\Omega}_{s2}$ propagated to the time t_m.

For the automatic mode, the decision function is as follows:

1) Preliminary correlated satellites are sought; they satisfy the conditions

$$\Delta\lambda = \min(|\lambda_{s1} - \lambda_m|_{\bmod 2\pi}, |\lambda_{s2} - \lambda_m|_{\bmod 2\pi}) < c_{\lambda a}$$

$$\Delta\Omega = \min(|\tilde{\Omega}_{s1} - \Omega_m|_{\bmod 2\pi}, |\tilde{\Omega}_{s2} - \Omega_m|_{\bmod 2\pi}) < c_{\Omega a}/i_s$$

$$\Delta i = \min(|i_{s1} - i_m|, |i_{s2} - i_m|) < c_{ia}$$

$$\Delta\tilde{i} = \min(|\tilde{i}_{s1} - i_m|, |\tilde{i}_{s2} - i_m|) < c_{ia}$$

$$\Delta T = \min(|T_{s1} - T_m|, |T_{s2} - T_m|) < c_{Ta} \tag{11}$$

where the first condition for i is tested only for those objects that maintain the inclination close to a specified value (normally, near zero) and the second condition is for active satellites that do not maintain the inclination (for these satellites, the evolution of this parameter is the same as for passive objects); $c_{\lambda a}$, c_{ia}, $c_{\Omega a}$, and c_{Ta} are the threshold values; $c_{\lambda a}$ depends on the inclination; and c_{Ta} depends on σ_{Tm}, σ_{Ts1}, and σ_{Ts2}.

2) For each preliminary correlated satellite, the functional is calculated. The equation is similar to Eq. (10). However, the constants c_{i2}, $c_{\Omega 2}$, and $c_{\lambda 2}$ are different.

3) The final correlation of the measurements is completed only when the preliminary correlation of all of the measurements acquired during the considered observation session of the sensor is completed. The decision making procedure is more sophisticated than the procedure used for passive satellites. This procedure coincides with the decision function used for correlation of Russian and American catalogs of LEO satellites.[8] We select the groups of measurements and objects, characterized by the following properties: 1) each measurement of the group of measurement correlates to at least one satellite from the group of satellites, 2) each satellite from the group of satellites correlates at least to one measurement from the group of measurements, and 3) each of the groups has minimal possible size.

The process of correlation with the group starts from the pair object–measurement, characterized by the minimal value of functional (10). The relevant correlation is completed, and this pair is excluded from further consideration. For the remaining measurements of the group, the process continues until either no uncorrelated measurements remain or each satellite of the group has obtained the measurements correlated with it. In the latter case, the remaining measurements are considered uncorrelated.

Regarding the described decision procedure, it is expedient to note the following. For passive satellites, a more simple procedure is used. It is efficient because the orbit determination and propagation errors (for the moment of the observation) for passive satellites virtually always are smaller than the distance between them.

For active satellites, the situation is different. The lack of complete information on the performed corrections leads to errors in the determination of their parameters (for the time of the observation), which are sometimes comparable or even exceed the distances between close active satellites. In this case, the decision procedure for active satellites provides better quality of the decision. For the most accomplished cases, when the alien satellite is closer to the measurement than the own one, the correct decision is possible only on the basis of either the data on the time and momentum of the performed corrections or additional information. The additional information here means the brightness curves of the satellite, which can be acquired by some optical–electronic sensors during the observation sessions. Currently, some stations already use these types of data in the correlation process.

Our work considers only the issues of track measurements processing. The issues of processing nontrack data are beyond the scope of the work and must be subjected to special investigation.

Note that sometimes the measurement is produced by an active cataloged satellite, but the correct correlation decision cannot be achieved. In this case, the measurement, as a rule, becomes uncorrelated and enters the preliminary tracking process. This is characteristic for the situations when the measurement is acquired after the performed maneuver and the data on the maneuver are lacking. The maneuver may be performed, for example, to transfer the satellite to another point of the GEO ring or to the graveyard orbit after the end of the active mission. Compared to correction, the momentum of the maneuver is significantly greater, and the gates, used for measurement–satellite correlation, are not suitable for such events. The technique of measurement–satellite correlation for this case is described in the "Preliminary Tracking" section.

Interactive Mode

To complete this section, we will discuss the issues of the capabilities of the interactive mode used for autonomous operations.

Autonomous works with participation of the analyst are performed in the situations in which the automatic procedures fail to produce decisions on measurement–satellite correlation with sufficient quality. The decision to carry out these works is made on the basis of the analysis of the calculations, fulfilled by automatic programs.

Autonomous works are carried out in the following situations: 1) the measurement is not correlated in the automatic mode; 2) the measurement is correlated to two satellites (passive and active); 3) correlation of measurements, obtained in the SSC, contradicts the correlation performed on-site; 4) the measurement is correlated to the satellite of preliminary tracking stage; 5) the residuals between the measurement and the orbit of the satellite, to which it is correlated, are too great; 6) the measurement has low accuracy, for example, comprises only one mark, or includes the marks that did not inscribe into the orbit, generated on their basis; and 7) no orbit is generated using the marks of the measurement. Note that, for situation 4, the measurements with different satellite numbers for the sensor are identified with one and the same satellite of the SSC catalog, or the number of the real satellite, for example, the international designator, is different from the number, determined by the SSC.

In general, the algorithm of preliminary correlation for the interactive mode is similar to the procedure used in the automatic mode.

The specific feature is that the values of the gate constants in Eqs. (9) and (11) can be chosen by the analyst, and their default values are greater than for the automatic mode. The analyst, having the residuals in all of the parameters displayed on the monitor together with the accuracy characteristics of the orbit and the measurement, makes the final choice of the satellite correlating to the measurement. In addition, at the request of the analyst, the mode of working with individual marks can be switched on. (This mode requires significant CPU time and, therefore, is recommended only for single marks and also for the measurements for which the automatic correlation failed.) This procedure is not available in the automatic mode. In this mode, the decisions are made for each mark separately. The certain mark is preliminary correlated to the satellites for which the following conditions are satisfied:

$$\Delta\alpha = |\alpha_{mj} - \alpha_{sj}|_{\bmod 2\pi} < c_\alpha$$

$$|\delta_{mj} - \delta_{sj}| < c_{\delta 1} + c_{\delta 2}(\Delta\alpha)i_s |\delta_{mj} - \tilde{\delta}_{sj}| < c_{\delta 1} \tag{12}$$

where α_{sj} and δ_{sj} are the result of propagation to the time t_{mj} (of the catalog orbit for passive satellites or the archived orbits for active satellites with subsequent transformation into parameters of jth mark); $\tilde{\delta}_{sj}$ is the result of propagation of satellite orbit to the time, closest to t_{mj}, for which the calculated value of parameter α coincides with α_{mj}; and c_α, $c_{\delta 1}$, and $c_{\delta 2}$ are constants. The final decision is made by the analyst.

As the result of autonomous work, the measurement either becomes correlated to a certain satellite, or remains uncorrelated. [In case the decision is based on the individual marks of the measurement, different decisions are possible for different marks. Some of the marks may become correlated (and correlation to one satellite is not obligatory), the others remain uncorrelated.] The uncorrelated measurements (marks) participate in preliminary tracking described in the "Preliminary Tracking" section.

Orbit Updating

The correlated measurement is recorded in the archive of data on the object: the orbit in the archive of orbits and the marks in the archive of marks. After that, the orbital parameters of the satellite are updated.

The functional minimized in the process of orbit updating is close to the functional used for updating of LEO orbits. However, the minimization procedure is essentially simpler. The explanation is as follows. On one hand, for the objects in GEO, the instability of the perturbations of the orbital elements and the prediction errors are significantly smaller. This situation makes the problem of estimating the orbital parameters more linear and, thus, more simple. On the other hand, the abnormalities in optical measurements are more rare than in radar measurements, and thus the procedure used to reveal them can be more simple.

Functional for Minimization

Parameters of the updated orbit minimize the functional

$$\Phi(\boldsymbol{a}) = \sum_{p=1}^{M}[x_p - f_p(\boldsymbol{a})]'(\boldsymbol{K}_p + \tilde{\boldsymbol{K}}_p)^{-1}[x_p - f_p(\boldsymbol{a})] + \sum_{q=1}^{N}[a_q - a_q(\boldsymbol{a})]'\left(\boldsymbol{K}_{a_q} + \tilde{\boldsymbol{K}}_{a_q}\right)^{-1}[a_q - a_q(\boldsymbol{a})] \qquad (13)$$

where $\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_M$ are optical measurements with timings $t_{1m} \leq t_{2m} \leq \cdots \leq t_{Mm}$ correlated to a specific object; $f_p(\boldsymbol{a})$ is the functional relationship between parameters of the observation $\boldsymbol{X}_p$ and parameters of the satellite $\boldsymbol{a}$; $(\boldsymbol{K}_p)_{ij} = \sigma^2_{x_{p,s}}\delta_{ij}$ where $\delta_{ij} = 0$ for $i \neq j$ and $\delta_{ij} = 1$ for $i = j$ and $\sigma^2_{x_{p,s}}$ is the mean square of the error of ith component $\boldsymbol{X}_p$, $i, j = 1, 2, \ldots, 6$. Also $(\tilde{K})_{ij} = \tilde{\sigma}^2_{x_{p,s}}\delta_{ij}$, where $\tilde{\sigma}^2_{x_{p,s}}$ is the mean square of predicting the orbit for the time t_{pm} (for parameter of ith component of $\boldsymbol{X}_p$), $i, j = 1, 2, \ldots, 6$; $\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_N$ are the orbital parameters acquired from other sources with timing $t_{1s} \leq t_{2s} \leq, \ldots, \leq t_{Ns}$; $\boldsymbol{a}_q(\boldsymbol{a})$ is the functional relationship between parameters $\boldsymbol{a}_q$ and satellite parameters $\boldsymbol{a}$; K_{a_q} is the correlation matrix of the errors for $\boldsymbol{a}_q$, structurally similar to $\boldsymbol{K}_p$; $\tilde{\boldsymbol{K}}_{a_q}$ is similar to $\tilde{\boldsymbol{K}}_p$; and superscripts $'$ and -1 denote matrix transposition and inversion, respectively.

Let us discuss the choice of the parameters of functional (13) in the automatic mode. Measurement parameters vector $\boldsymbol{X}_p$ is a six-dimensional vector $(D_p, \alpha_p, \delta_p, \dot{D}_p, \dot{\alpha}_p, \dot{\delta}_p)$, resulting from the orbit determination using the marks of the measurement. The orbital parameters vector $\boldsymbol{a}$ is a seven-dimensional vector $\boldsymbol{a} = (\tilde{\lambda}, L, p, q, \tilde{h}, \tilde{k}, s)$, where $\tilde{\lambda}$, L, p, q, $\tilde{h}$, and $\tilde{k}$ are orbital elements (see the "Primary Determination of Orbits" section) and s is the average coefficient of light pressure. Vector $\boldsymbol{a}$ is referred to the time $\max(t_{Mm}, t_{Ns})$.

The variances of the errors $\alpha^2_{x_{p,s}}$ of the components of each measurement are obtained in the process of minimization (along with determination of the primary orbit). Certain constants pose the lower limits for the calculated values for all parameters of the measurement [diagonal elements of the matrix inverse to the matrix of second partial derivatives of functional (3)]. The variances of errors in the components of $\boldsymbol{a}_q$ (diagonal elements of matrix $\boldsymbol{K}_{a_q}$) are constants, chosen empirically for each data source individually.

The diagonal elements of matrices $\tilde{\boldsymbol{K}}_p$ and $\tilde{\boldsymbol{K}}_{a_q}$, which characterize prediction errors, were also chosen experimentally. The residuals between the observed and calculated using the updated orbit parameters for various cataloged satellites were the basis for the analysis. The result of the analysis is specific functions, chosen for all of the parameters. These functions are expressed by a combination of polynomials, are equal to zero for the zero propagation interval, and do not decrease with the increase of propagation interval. The orders of the polynomials for parameters D, δ, $\dot{D}$, $\dot{\alpha}$, and $\dot{\delta}$ do not exceed the first and for parameter α the second.

The time interval $\Delta t = \max(t_{Mm}, t_{Ns}) - \min(t_{1m}, t_{1s})$ used to retrieve the orbits of passive satellites from the archive is 500 days. For active satellites, the updated values of the orbital parameters are the parameters of the last orbit from the archive.

Minimization of the Functional

The classical Gauss–Newton technique is used for minimization of functional (13). This technique implies that one iteration must be performed in search of the minimum of Eq. (13). The updated vector of orbital parameters $\boldsymbol{a}_{up}$ is calculated using

$$\boldsymbol{a}_{up} = \boldsymbol{a}_{pr} - \left[\mathrm{d}^2\Phi(\boldsymbol{a}_{pr})\right]^{-1}\mathrm{d}\Phi(\boldsymbol{a}_{pr}) \qquad (14)$$

where $\boldsymbol{a}_{pr}$ are orbital parameters, obtained after the last updating, predicted (extrapolated) to the time $\max(t_{Mm}, t_{Ns})$ and $\mathrm{d}\Phi(\boldsymbol{a}_{pr})$ and $\mathrm{d}^2\Phi(\boldsymbol{a}_{pr})$ are gradient vector and the matrix of the second partial derivatives for the minimized functional $\Phi(\boldsymbol{a})$ in the point $\boldsymbol{a}_{pr}$, which are equal to

$$\mathrm{d}\Phi(\boldsymbol{a}_{pr}) = -\sum_{p=1}^{M}\mathrm{d}\boldsymbol{f}_p(\boldsymbol{a}_{pr})'(\boldsymbol{K}_p + \tilde{\boldsymbol{K}}_p)^{-1}[x_p - f_p(\boldsymbol{a}_{pr})] + \sum_{q-1}^{N}\mathrm{d}\boldsymbol{a}_q(\boldsymbol{a}_{pr})'\left(\boldsymbol{K}_{a_q} + \tilde{\boldsymbol{K}}_{a_q}\right)^{-1}[\boldsymbol{a}_q - \boldsymbol{a}_q(\boldsymbol{a}_{pr})]$$

$$\mathrm{d}^2\Phi(\boldsymbol{a}_{pr}) = \sum_{p=1}^{M}\mathrm{d}\boldsymbol{f}_p(\boldsymbol{a}_{pr})'(\boldsymbol{K}_p + \tilde{\boldsymbol{K}}_p)^{-1}\mathrm{d}f_p(\boldsymbol{a}_{pr}) + \sum_{q=1}^{N}\mathrm{d}\boldsymbol{a}_q(\boldsymbol{a}_{pr})'\left(\boldsymbol{K}_{a_q} + \tilde{\boldsymbol{K}}_{a_q}\right)^{-1}\mathrm{d}\boldsymbol{a}_q(\boldsymbol{a}_{pr}) \qquad (15)$$

[Note that, in the formula for $\mathrm{d}^2\Phi(\boldsymbol{a}_{pr})$, there is no account of the terms comprising residuals $\boldsymbol{x}_p - \boldsymbol{f}_p(\boldsymbol{a}_{pr})$ and $\boldsymbol{a}_q - \boldsymbol{a}_q(\boldsymbol{a}_{pr})$.] The matrices $\mathrm{d}\boldsymbol{f}_p(\boldsymbol{a}_{pr})$ and $\mathrm{d}\boldsymbol{a}_q(\boldsymbol{a}_{pr})$ of the partial derivatives of parameters $\boldsymbol{x}_p$ and $\boldsymbol{a}_q$ with respect to parameters $\boldsymbol{a}$ are calculated using a finite differences technique.

Automatic Mode

Here we discuss the organization of the computation process for orbit updating in the automatic mode.

Before updating, the weights of the components, that is, coefficients of the squares of the residuals in $\Phi(\boldsymbol{a})$, are chosen for $\boldsymbol{x}_p$ and $\boldsymbol{a}_q$ (we will call them measurements). This is performed in the following way.

We select the components of the new (not previously participated in orbit updating) measurements that have significant errors, compared to the thresholds of the input residuals [the residuals, obtained before updating, in distinction from the output residuals, obtained after minimization of $\boldsymbol{\Phi}(\boldsymbol{a})$]. The weights of the selected components are nullified. The other components of the new measurements participate in the minimization with their weights calculated using Eq. (13) assuming that they are not abnormal.

All of the old (previously participated in the updating) measurements are used with the weights obtained after the previous updating, that is, the weights of the components of the measurements that were characterized by significant residuals during the last updating are nullified.

After that, the updating is performed using the algorithm described in the "Minimization of the Functional" section. The obtained result is tested for reliability. The solution is considered reliable when at least three measurements, including at least one new have inscribed (have acceptable residuals for all parameters) into the orbit.

If the new measurements did not inscribe into the generated orbit, an attempt is made to construct the updated orbit for smaller time interval (150 days).

When all of the performed operations have failed to produce a reliable orbit, the autonomous work using interactive mode of the updating program is performed.

Interactive Mode

In this mode, the input residuals between the last reliable updated orbit and the orbits from the archive with their timings and accuracies are displayed for the analyst.

On the basis of these data, the analyst chooses the measurements to calculate the updated orbit and the weights of the individual components of these measurements. Note that there are no limitations on the choice of the weights. In particular, the chosen values of the weights may correspond to the weights, characterizing the orbit, obtained using one mark (equal to zero for all components of the measurement except α and δ). The time interval for the measurements selected for orbit determination is not limited.

When the updating is completed, the output residuals are displayed instead of the input ones. Analysis is performed again. If the analyst considers that the updated solution is better than the initial one, this solution can be introduced into the catalog instead of the old updated orbit. Then the process of selecting the measurements and setting the weights can be resumed.

In the course of these works, the analyst can make a decision that some of the measurements are alien, that is, produced by other satellites. These measurements are removed from the archive of the object and are recorded in the catalog as uncorrelated. Their further processing is quite similar to the processing of other uncorrelated measurements (see the “Interactive Mode” section and the “Preliminary Tracking” section).

Planning of the Observations and Calculation of Target Indications

This algorithm has no analog in the maintenance of LEO satellite catalog because there is no control of radar sensors emanating from the SSC. For several reasons, currently this algorithm is not worked out completely. Thus, the share of autonomous work is rather significant here.

The planning of measurements is performed by the analyst. The principles of planning the observations for the purposes of GEO monitoring are as follows: 1) For passive satellites with an accurately determined orbit, maintenance of tracking requires 1–2 measurements satisfying the requirements of the SSC measurements per year. 2) For passive satellites when the orbit is not determined sufficiently accurate, the measurement is needed when the calculated value of the error of position determination is comparable or exceeds the maximal acceptable error of target indication. 3) For active satellites, the measurements are needed for the times when the change of the longitude of satellite projection or the orbital plane exceeds the usual limits for the satellite. Such situations occur when maneuvers are performed to transfer the satellite to the other orbit or to the point within GEO.

In addition, the observations are planned when launches or emergency situations occur. The purposes of satellite identification using nontrack data are also taken into account. Plans are prepared for each sensor individually taking into account the schedule of the works conducted on the site. Planning the work of a specific sensor, the analyst uses the data on the informational capabilities of this sensor, in particular, operation mode (photo or optical–electronic), penetrating capability, capacity, and a set of other factors, important for the work.

Selection of the satellites is performed on the basis of the catalog of the objects that will reside in the field of view of the sensor during the anticipated interval of observations. This catalog is called the partial catalog. The satellites in the partial catalog are prioritized according to the mentioned principles. Priorities of different groups of satellites may be different for different works, and, therefore, the analyst can choose the class of objects related to the particular job (for example, only active satellites with some properties or passive satellites with poor accuracy of current orbit determination).

The following types of data on the selected objects can be forwarded to an optical–electronic sensor: either target indications for individual satellites or partial catalogs for certain space domains. Photosensors receive the spatial zones where the required (for the SSC) satellites reside. (The SSC may not need all of the satellites within these zones. However, under the favorable conditions the sensor will acquire the measurements on all of the objects present in the photoplate.)

Let us discuss the algorithm for the calculation of the target indications and partial catalogs. For passive satellites, the algorithm is reduced to propagation of the parameters of the satellite and the required transformation into agreed form. For active satellites, the propagation is also fulfilled, but the obtained values of parameters $\tilde{\lambda}$, p, and q are replaced by the values providing that the longitude of satellite projection λ and orbital inclination i for the time of the observation t_{ob} will be equal to anticipated for this satellite values $\hat{\lambda}(t_{ob})$ and $\hat{i}(t_{ob})$. The estimations $\hat{\lambda}$ and $\hat{i}$ are obtained using averaging or approximating of the archive data. Averaging is used for the inclination of those satellites that maintain it; approximation is used for longitude of satellite projection for all active objects. It is considered that the function $\lambda(t)$ describing the evolution of the longitude of satellite projection for the moments of passing through the perigee is continuous and can be represented as the composition of linear functions within certain intervals.

Preliminary Tracking

After the processing of the measurements, some may still not be correlated with the cataloged satellites. These measurements are incorporated into the catalog as new objects for which the process of preliminary tracking begins.

In the course of preliminary tracking, the object is under the permanent control of the analyst. This analysis may lead to the following decisions: 1) the object is identified with the satellite already present in the catalog, 2) the object is considered unreliable and is removed from the catalog, and 3) the object is accepted as a new one not previously cataloged.

In the latter case, satellite identification must be fulfilled, that is, its origin (international designator) must be determined. Then the preliminary tracking is finished and the satellite is transitioned to regular tracking, for which the basic aspects are described in the “Primary Determination of Orbits” through the “Planning of the Observations and Calculation of Target Indications” sections.

The preliminary tracking of a satellite may last as long as several years. Let us consider the operations of the analyst.

The interactive program, calculating the approaches of the satellite to any cataloged object within a specified time interval, is used for identification. The principles of such a program are described in Ref. 9. The results of the calculations sometimes result in orbital identification for the cases of maneuvers, even when the data on the maneuver are not available.

When any data received and processed by the SSC can influence (according to the analyst) the destiny of a preliminary tracked satellite, an attempt to identify this object with the other ones is made. The interactive version of the measurement–satellite correlation program is used. Sometimes the attempt is successful. Let us consider a typical situation. The SSC received a measurement on the satellite that performed a maneuver. The measurement is received at the time when the satellite is already rather far (regarding the longitude) from the point of its residence before the maneuver. For the moment of processing this measurement, no information on the maneuver was available for the SSC. Thus, the measurement remained uncorrelated. Some time later, the SSC receives the data on the maneuver. Then the repeated run of the correlation program in interactive mode leads to the required identification.

Target indications are forwarded to the optical sensors on preliminary tracked satellites to confirm the existence of the satellite in orbit. In case we are unable to identify the object with the cataloged satellites, but its residence in orbit is confirmed by at least two sensors, the object is considered to be a new satellite.

We will not discuss the procedures used for complete identification of new satellites because rather often their essence is not within the scope of orbital monitoring. We will just note that for satellite identification the nontrack data of optical sensors are used as well as the data on satellite coordinates. The data from other sources of information are used also.

Results of Trial Operations

We will now consider some of the results obtained during testing and trial operations of the described complex of the algorithms for orbital monitoring of GEO satellites.

Table 2 Orbit determination and prediction errors

t_{pr}, days	$k_{0.5}$, °(km)	$k_{0.8}$, °(km)	$k_{0.9}$, °(km)
0–100	0.016 (10)	0.035 (22)	0.050 (32)
100–200	0.015 (9.5)	0.033 (21)	0.047 (30)
200–300	0.016 (10)	0.035 (22)	0.058 (36)
300–400	0.014 (8.8)	0.031 (20)	0.065 (41)
400–500	0.019 (12)	0.047 (30)	0.085 (54)
500–600	0.038 (24)	0.090 (57)	0.17 (110)
600–700	0.052 (33)	0.14 (88)	0.22 (140)
700–800	0.080 (50)	0.18 (110)	0.32 (200)
800–900	0.10 (63)	0.27 (170)	0.40 (250)
900–1000	0.13 (82)	0.30 (190)	0.43 (270)
1000–1100	0.16 (100)	0.39 (250)	0.62 (390)
1100–1200	0.20 (130)	0.49 (310)	0.70 (440)
1200–1300	0.22 (140)	0.50 (310)	0.75 (470)
1300–1400	0.26 (160)	0.60 (380)	1.0 (630)
1400–1500	0.30 (190)	0.70 (440)	1.3 (820)

1) Real errors of orbit determination and propagation are the basic characteristics because they determine the overall performance of the catalog maintenance algorithm and the characteristics of its components.

The technique used for assessment of the real errors of orbit determination and prediction is as follows. For each passive satellite, the residuals between its last updated orbit and all of the archive orbits were calculated for an interval of 4 years. The distributions of these residuals for all passive satellites in the catalog were constructed for 15 propagation intervals t_{pr} (0–100 days, 100–200 days, etc.). The quantities for the distributions of absolute values $k_{0.5}$, $k_{0.8}$, and $k_{0.9}$ were calculated for the levels 0.5, 0.8 and 0.9. Table 2 presents the results for the parameter $\tilde{\lambda}$. Note that this is the upper estimation of the real errors along the track of a satellite, that is, the maximal errors in determination of the predicted position of the object. With the increase of propagation interval, this estimation approaches the estimation of real errors. One can see from Table 2 a) for prediction intervals up to 500 days, the errors of position determination do not exceed $1'$ (12 km in linear values) for 50% of cases, b) for prediction intervals up to 1500 days, the errors do not exceed 0.3° (190 km) for 50% of cases, and c) for 80% (90%) of cases, the errors are 2–2.5 (3–4) times greater than for 50% of cases.

2) The main characteristic of the program of primary determination of orbits is the percentage of measurements from which the reliable orbit can be generated. Now we have achieved virtually 100% level for automatic production of reliable orbits on the basis of measurements of optical–electronic sensors. For the photosensors, this parameter is lower. The major obstacle is the presence of marks from different objects in the measurement.

3) Correlation of measurements with the cataloged satellites is characterized by the share of the correlated measurements. The results of data processing in 1997 yield that 91% of satisfactory measurements are correlated automatically, and an additional 7% of these measurements are correlated using autonomous work. Thus, approximately 2% of satisfactory measurements become uncorrelated. The measurement is considered satisfactory in case it contains not less than two marks within the interval not less than 2 min and the reliable orbit was generated on its basis. When only this measurement is used, it is possible to determine not less than four orbital parameters out of six with accuracy, sufficient for catalogization.

4) The majority of the uncorrelated measurements are either of low accuracy (all of the marks are acquired within the interval shorter than 0.3 h or there are only two of them) or are generated by the object whose parameters do not satisfy Eq. (1). These measurements are then removed from the catalog. Four uncorrelated measurements acquired in 1997 are left in the catalog as preliminary tracked objects. All of them still (September 1998) remain in this position.

Conclusions

1) Development of the complex of algorithms for maintenance of the catalog of GEO satellites incorporated all of the experience of solving this task for LEO satellites. Thus, a simple and efficient system was created.

2) The initial data, that is, the characteristics of the sensors and the satellites, determine the structure of the catalog maintenance algorithms. The most important are the following distinctive features of the initial data for GEO objects: a) optical principle of data acquisition, b) possibility to control the sensors from the SSC, c) low average density of GEO satellites and the flux of measurements, d) small effect of unstable perturbations on the motion of satellites, and e) significantly greater number (absolute and relative) of satellites performing orbital maneuvers and corrections.

3) Similar to the LEO region, several essential limitations for the range of observed parameters, the rate and accuracy of the observations exist for GEO satellites as well. Therefore, the catalog maintenance algorithm uses all available data that are stored in the historical archives, and, thus, the informational losses are not too high.

4) The employed program for predicting the motion of passive satellites of the GEO domain has acceptable accuracy characteristics and very high computation rate. Thus, the process of tracking and detection was essentially simplified compared to similar process for LEO satellites.

5) The specific features of initial data led to the increased, compared to LEO satellite data processing, share of autonomous works using interactive modes. This is especially characteristic for the detection process, which, in fact, is performed autonomously. This mode is also used for satellite tracking when orbital maneuvers occur. Planning of the work of sensors and calculation of target indications also includes manual operations. We hope that in the future the work with the sensors will be performed automatically. At present, it is not possible due to several issues not related to the scope of the present paper.

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